Phase Space Planning and Optimization of Foot Placements in Rough Planar Terrains

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Abstract— **Operating and maneuvering in difficult terrains has remained a challenging problem in the field of legged robots. One of the major challenges arises from the high dimensionality inherent in planning foot placements coupled with center of mass motion along terrains that are multifaceted and highly diverse. Previous work has resolved these issues to an extent by constraining the center of mass to fixed trajectories or using predetermined foot placements. To deal with these challenges, this paper proposes a new set of strategies: (1) an optimized geometric Hermite curve with minimum curvature and length is used to plan the motion of the center of mass (2) single contact model dynamics for state-space approximations of center of mass behavior are used to resolve feet transitions between steps in planar environments and (3) vertical center of mass phase space trajectories are optimized to produce an overall plan with minimum energy. This framework allows us to synthesize complex maneuvers in rough terrains and to develop optimal contact transition and foot placement plans that consider the robot's configuration and constraints. Experimental results show that for any potential locations of foot contacts, our planner generates smooth and optimal trajectories for center of mass motion as well as a minimum energy plan for transitioning between foot placements.**

I. Introduction

The planning and sequencing of foot placements in rough and complicated terrains has remained an important goal in the development of motion plans for legged robots. Given the cyclopean space of natural terrains and the prodigious number of available motion plans that exist therein, it is evident that creating locomotion plans that balance successful navigation with foot placement is a very difficult task. In nature, solutions to the problem have evolved to produce distinct taxonomical characteristics and diversity in patterns of movement among animals [1]. Indeed, an animal's performance in locomotion is directly tied to its survivability, and thus developing an efficient motion plan takes on the utmost importance: some animals prefer speed or acceleration and have evolved long and nimble limbs for increased flexibility in feet and body placement. Others favor stability and security of foot contact, and have evolved sturdy and stumpy limbs [1]. Even within a single species, the range of physical sizes and shapes is so varied that creating models that unify their physical representations is a daunting task. Indeed, the enormous scale of size of physical shapes and locomotor mechanisms coupled with the breath of environments possible conspire to make the problem highly intractable [2]. But it is precisely these criteria that must be considered when planning across complex terrains.

Robots, like their biological brethren, also suffer from the same enormity of scale. Like animals and humans, they also come in myriad shapes and configurations. Bipedal robots, for instance, have been designed to function like humans and also bear the same magnitude of disparity in shapes as their human counterparts. Furthermore, to function like humans, these robots must achieve the same level maneuverability and economy of motion, and must do so at slow or high speeds. Humans alone are capable of maneuvering across extreme terrains, performing a wide range of motions while varying speed, direction and contact locations. These complex maneuvers are governed by nonlinear dynamics that lack a closed form solution and thus make extraction of behaviors very difficult. Specifically, determining contact transitions and foot placements across varying terrains while traveling at different speeds has proved moiling for motion planners to incorporate. If appropriate models and strategies can be developed, then their application in motion planning and control could open up new avenues in the development of extreme locomotion maneuvers for robotic systems. Perforce, developing models and control policies of the dynamic behavior of motion of a robot is extremely consequential in planning and navigating through complex terrains.

In this paper, we address the issues of motion planning and foot placement selection by creating general strategies from which diverse robot configurations can be analyzed and modeled. Our contribution lies in developing a framework from which foot placements and center of mass motion plans can be produced that incorporate the robot's configuration as well as characteristics of the terrain. The plans developed yield optimal trajectories for the robot's center of mass as it travels over the environment as well as optimizing the contact foot locations in the terrain. With these strategies in place, we aim develop locomotion plans that more closely resemble the dynamic and extreme motions of animals and humans. These models can then be used to plan and control robots of different configurations traveling through diverse environments.

II. Related Work

Our objective is to develop center of mass(CoM) motion and foot placement plans that take into account the robot's configuration and characteristics of the terrain. The hallmarks of our approach are the ability to generate CoM paths directly. This is done by creating abstractions of the robot's configuration. The non-reliance on given CoM paths provides greater flexibility in planning across robots with different kinematic constraints. Additionally, characteristics of the terrain are used to optimize the locations of foot contacts to produce minimum energy foot plans. The simultaneous study of CoM planning and optimal foot placement selection has so far not been treated, especially not with respect to rough environments. Previous work has been limited in its scope and lacked the malleability to deal with these terrains.

Limit cycle based techniques were introduced in [3] where control of foot placement is implicit and the result of passive dynamic walking. Because the gait is powered naturally by gravity, passive limit cycle based robots are stable to small perturbations and require less energy. Although these techniques provide stable walking, their passive nature disallows CoM planning and restrict selection of foot placements. Moreover, these techniques have shown poor performance on rough terrains. Such terrains are replete with varying heights which require the ability to plan CoM paths and contact foot placements for successful transit. To compensate, further research has immixed biological elements with limit cycle techniques. In [4] and [5] artificial muscles and foot model analysis was investigated to enlarge the stable walking range of biped robots. However these methods still lack tolerance to rough terrains and offer crude CoM and foot placement planning.

Another popular approach is Zero Moment Point based control. In ZMP the robot's configuration is adapted to satisfy predefined foot placement locations [6]. The drawback of such methods lies in their restriction of the robot to limited motions that meet the foot placement conditions. Consequently, robots implementing ZMP based control lack robustness against perturbations, demonstrate poor performance on uneven terrains, and offer reduced energy efficiency. Thus, ZMP research has focused on methods that improve adaptivity such as auxiliary ZMP based control [7] and adaptation of reference trajectories to large disturbances [8] [9] [10]. In [11] and [12], the authors simultaneously planned CoM and ZMP trajectories using parametrized polynomials and handled small modifications of foot placement. However, the CoM path was calculated only indirectly as a result of the ZMP path and the foot contact locations were also preplanned in advance. Such methods limit the choice of CoM path and perform no discriminatory paring of contact foot locations resulting in reduced flexibility over terrains.

Lastly, Capturability based approaches use gait models to approximate capture regions that allow planning of foot contacts [13] [14] [15]. Although effective, the simplified gait models developed have only been demonstrated to work on level terrains. Furthermore, the simplifications presented discard valuable information, such as height variations of the CoM during locomotion. In contrast, our method handles uneven terrains and accounts for CoM height changes, thus generalizing to larger classes of environments and robots.

More recently, the authors in [16] [17] [18] [19] have made significant contributions in understanding the multi-contact dynamics of extreme locomotion maneuvers in rough terrains. Our work utilizes and extends the models they developed to include planning of CoM paths that are contingent upon the robot's kinematics and the terrain being traversed to create optimized contact foot transitions and foot placements. Unlike all the previously mentioned work and in particular that given in [11], [14] and [21], our framework allows greater freedom in the selection of both the CoM path and the foot contact locations. CoM paths that respect the unique configuration of the robot can be specified directly. These in turn are used as a guideline to find optimal foot locations and contact transitions in the environment. With this framework in hand, we can effectuate the locomotion plan across variable terrain for any robot.

III. Overview of Framework

CoM path and optimal contact foot placement planning is achieved as follows: (1) an optimized geometric Hermite curve with minimum curvature and length is used to plan a path for the robot's CoM. The curve satisfies conditions dependent on the robot's configuration, and thus allows us to encapsulate the ideas of size and functional variation among different robots. (2) Inverted pendulum dynamics and perturbation theory are used to obtain phase curves of CoM behavior. The behavior of the CoM is critically dependent on the foot contact locations. The foot contact locations themselves are conditional on the environment being traversed. (3) This dependency is exploited to generate minimum energy cost landscapes of vertical CoM phase behavior. (4) Through iteration of the ensuing foot across the terrain, the optimal foot location is chosen from the cost landscape. Selection is made to meet the kinematic constraints of the robot. (5) The process is repeated starting from the preceding foot location to generate successive contact locations. In this manner, optimal foot placement and CoM path plans are accomplished. The overall framework is shown below.

Algorithm 1 Optimal CoM and Foot Placement Planner

1. Given a robot configuration k and a terrain q, determine the range of viable CoM vertical positions r, for k in q.
2. Initialize first foot placement F_0 and CoM position P_0 and tangent vector V_0.

loop

  3. Using Hermite endpoints P_i, V_i $i = 1...n$, $P_i \in r$ representing CoM position and tangent vector, generate optimal geometric Hermite curves with minimum curvature $g_i(x)$, $i = 1...n$.
  4. Select the curve $g_i(x)$ with minimum length.
  5. Let CoM path be $g_i(x)$.
  6. Select candidate secondary foot placement F_i.

  for each pair of F_0 and potential F_i **do**

    a. Use $g_i(x)$ to find combined CoM phase curve and contact transition point for F_0 and F_i.
    b. Find minimum strain energy cost, C_i of resulting phase curve.
    c. Choose next foot placement candidate F_i by iterating along the terrain q.
    d. Select F_i yielding the lowest cost C_i.

  end for

  7. $F_0 = F_i$, $P_0 = P_i$, $V_0 = V_i$

end loop

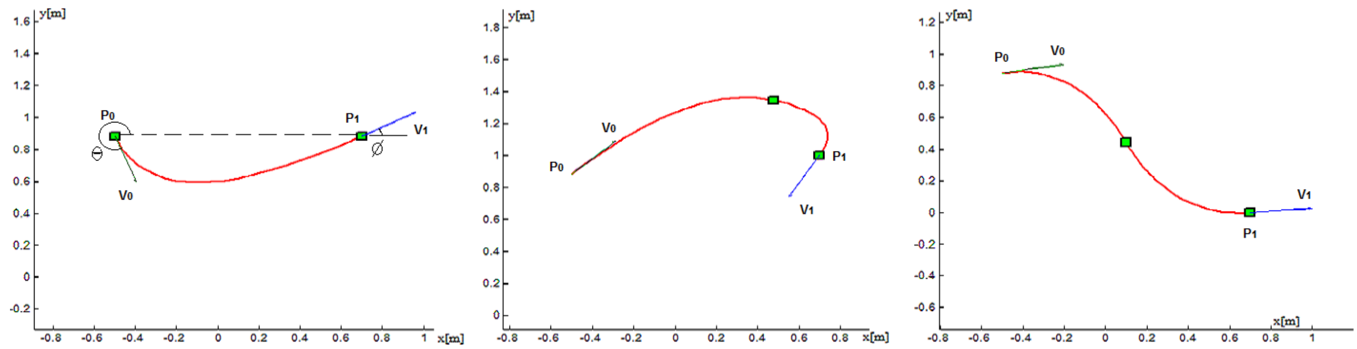

Fig. 1. OGH curves with different vector tangent angles. The center and right plots show COH curves constructed using piecewise OGH segments. The angles θ and ϕ are both measured counterclockwise with respect to the vector $\overline{P_0P_1}$, with θ corresponding to the point P_0 and ϕ to P_1. Note the diversity of complex yet smooth paths possible.

A. *Optimized geometric Hermite curve*

The pivotal element in robot locomotion essentializes to the ability to maintain control over the position and velocity of the robot's center of mass. The breadth of viable CoM paths is vast but depends critically on the terrain the robot is traversing and the foot placements therein. CoM dynamics can be analyzed by studying the interaction and interplay between these components. In this paper, we use an optimized geometric Hermite(OGH) curve to plan CoM paths through a terrain. OGH curves lend themselves well to CoM path planning because they are both mathematically and geometrically smooth. In addition, the endpoint tangent vectors can be specified, thus allowing for directional and velocity control of CoM paths.

OGH curves have been optimized using various criteria including: minimum length, minimum strain energy and minimum curvature [22] [23] [24]. Here we use OGH curves with minimum curvature first defined in [24] since they render smooth and fluid curves that are often the required basis for planning over variable terrain.

Definition 1. Given two endpoints P_0 and P_1, and two endpoint tangent vectors V_0 and V_1, a cubic polynomial curve $P(t)$, $t \in [t_0, t_1]$, is called an optimized geometric Hermite(OGH) curve with respect to the endpoint conditions P_0, P_1, V_0, V_1 if it has the smallest curvature variation among all cubic Hermite curves $\overline{P}(t)$, $t \in [t_0, t_1]$ satisfying the following condition:

$$\overline{P}(t_0) = P_0, \overline{P}(t_1) = P_1, \overline{P}'(t_0) = \alpha_0 V_0, \overline{P}'(t_1) = \alpha_1 V_1 \quad (1)$$

where α_0 and α_1 are arbitrary real numbers, $\overline{P}'(t)$ is the first derivative of $\overline{P}(t)$, and the cubic Hermite curve $\overline{P}(t), t \in [t_0, t_1]$ satisfying the constraints in (1) can be expressed as

$$\begin{aligned}\overline{P}(t) = (2s+1)(s-1)^2 P_0 + (-2s+3)s^2 P_1 + \\ (1-s)^2 s(t_1 - t_0)\alpha_0 V_0 + (s-1)s^2(t_1 - t_0)\alpha_1 V_1\end{aligned} \quad (2)$$

where $s = (t - t_0)/(t_1 - t_0)$. The objective function to optimize is defined as

$$E = \int_{t_0}^{t_1} [\overline{P}'''(t)]^2 dt \quad (3)$$

and is the approximate curvature variation of the curve $\overline{P}(t)$.

The tangent angle requirements are specified with respect to θ and ϕ where θ is the counterclockwise angle from the vector $\overline{P_0P_1}$ to V_0, ϕ is the counterclockwise angle from the vector $\overline{P_0P_1}$ to V_1, and θ and ϕ are called the tangent angles.

To guarantee both mathematical and geometric smoothness, the tangent angles must meet angle constraints. For angles outside these regions, a composite optimized geometric Hermite(COH) curve is constructed. A COH is a piecewise cubic polynomial composed of 2 or 3 segment pieces wherein each piece is an OGH curve. For complete details regarding angle constraints and construction of OGH and COH curves, see [24]. Fig 1 above depicts several OGH curves.

B. *Single contact point model*

The OGH curve defined above lays the groundwork for planning smooth CoM paths over volatile and inconstant terrains. However, the behavior(state space trajectory) of the CoM is fundamentally reliant on both the CoM path and the locations of the foot contacts. As the robot maneuvers and negotiates its environment, it makes contact with the terrain via its feet, which determine the overall dynamics of the system. In this study we treat the single contact point support paradigm explicated in [16] [17] [18] to gain insight into the dynamic behavior of the CoM under these conditions. The principle of dynamic equilibrium states that the sum of all moments acting on a moving system is equal to the net inertial moment. For the single contact paradigm, the balance of moments is given by:

$$p_{cop_k} \times f_{r_k} = p_{com} \times (f_{com} + Mg) + m_{com} \quad (4)$$

where k is the contact limb, p_{cop_k} is the limb's center of pressure(CoP) or contact point, M is the total mass of the system, g is the gravitational constant, p_{com} is the location of the CoM with respect to the coordinate origin, and f_{r_k}, f_{com}, and m_{com} are the 3-dimensional vectors of reaction forces, center of mass inertial forces and moments, respectively. The dynamic equilibrium of forces under the single contact model can be expressed as $f_{r_k} = f_{com} + Mg$ and in vectorial form in which the inverted pendulum model is assumed to be a single point mass such that inertial moments about the center of mass can be ignored, i.e., $m_{com} = 0$ [7] [20]. If we consider a planar model consisting of only the vertical and frontal directions [17] [18] and use Newton's second law to express force as a function of acceleration and mass, namely, $f_{r_{[kx]}} = Ma_{com[x]}$, $f_{r_{[ky]}} = Ma_{com[y]}$, and $f_{r_{[kz]}} = M(a_{com[z]} + g)$, equation (4) can be rewritten as:

$$a_{com[x]} = \frac{(p_{com[x]} - p_{cop_k[x]})(a_{com[z]} + g)}{p_{com[z]} - p_{cop_k[z]}} \quad (5)$$

for the dynamic behavior of the CoM in the sagittal direction. Similar expressions can be obtained for the lateral and vertical directions.

C. State space behavior

Equation (5) gives the acceleration profile of the CoM as a function of the position of the CoM and the position of the foot contacts. As discussed in [16] [17], the trick is to find a dependency by seeding a preselected geometric CoM path such that a_{com_x} can be represented as a function of only p_{com_x}, p_{cop_x}, and p_{cop_z}. Under such circumstances, the general form of equation (5) reduces to:

$$a_{com_x} = (p_{com_x} - p_{cop_{kx}}) \cdot \Phi(p_{com_x}, v_{com_x}, p_{cop_{kx}}, p_{cop_{kz}}) \quad (6)$$

where $\Phi(\cdot,\cdot,\cdot,\cdot)$ is a non-linear function and has no closed form solution. In our studies we use an OGH curve with minimum curvature constrained by the robot's configuration to determine the preselected geometric CoM path. Now using perturbation theory and supposing $x \triangleq p_{com_{[x]}}$ and $\ddot{x} \triangleq a_{com_x}$, equation(6) can be represented as $\ddot{x} = f(x, \dot{x})$. Over small iterations of x, the velocity and the acceleration of the CoM($\dot{x}$ and $\ddot{x}$ respectively) remain constant and can be used to approximate the behavior at neighboring points, i.e., CoM position versus its velocity(state-space trajectory). Thus, given initial conditions for $(x_k, \dot{x}_k)$, the change in CoM position over a time-step ϵ,(the perturbation), can be approximated as:

$$\dot{x}_{k+1} \approx \dot{x}_k + \ddot{x}_k \epsilon \quad (7)$$

$$x_{k+1} \approx x_k + \dot{x}\epsilon_k + 0.5\ddot{x}_k \epsilon^2 \quad (8)$$

Equation (7) can be used to find an expression for the perturbation in terms of velocity and acceleration to give $\epsilon \approx (\dot{x}_{k+1} - \dot{x}_k)/(\ddot{x}_k)$ which results in:

$$x_{k+1} \approx \frac{(\dot{x}_{k+1}^2 - \dot{x}_k^2)}{2f(x_k, \dot{x}_k)} + x_k \quad (9)$$

which is the state-space approximate solution we seek. For a thorough description outlining the process for finding state-space trajectories for the CoM, see [16] [17] [18].

IV. Path Planning

In the previous sections anent preselection of CoM paths, we discussed creation of the initial CoM path as being dependent on the robot's configuration as well as the environment being traversed. Clearly, the robot's kinematic constraints place restrictions on viable CoM positions, as does the environment which channels the robot along a confined space of paths. To characterize the kinematic variation among robots, we abstract the idea of a robot's ability to alter its CoM position into an interval range $\{(z_{min}, z_{max}) | z_{min} \le p_{com_z} \le z_{max}\}$, which represents the space of realizable CoM positions for a specific robot. Forsooth, this range will vary from robot to robot such that the paths generated will be unique, e.g. a small robot kinematically constrained to travel closer to the ground will have a range of viable CoM paths propinquant with the terrain, while a larger robot will have them further removed. This interval range is taken directly from the robot's kinematic constraints, i.e. leg lengths, joint limits etc, and in essence describes the robot's ability to shift its CoM position vertically. Additionally, in this study we treat only planar environments and thus (z_{min}, z_{max}) will depend on the terrain such that $\forall p_{ter_z} \in \mathbb{R} : |z_{min} - p_{ter_z}| = c$, where p_{ter_z} is the vertical position of the terrain and c is a constant. In other words, for each point in the terrain, there is a minimum threshold determined by the robot's kinematic constraints from which the CoM position is disallowed to enter. Using this range, we can create a family of OGH curves to plan CoM paths through the terrain. The process for performing this is as follows: (1) the interval range for the robot is determined, (2) the endpoint tangent vectors are specified. These are θ and ϕ explained previously and specify the direction the CoM should be traveling in at particular locations over the terrain. (3) For these locations, OGH/COH curves are traced out for each p_{com_z} in (z_{min}, z_{max}). If at any point in its length the OGH/COH curve exceeds the permissible CoM limits, it is rejected. This ensures that the CoM path respects the conditions of the environment and stays within the limits of the robot's configuration. (4) The curve with minimum length is selected from the family of curves generated. This yields a path with minimum curvature and length. Fig 2 below shows various OGH/COH CoM paths. The left and middle plot show example curves generated for separate tangent angle, robot CoM interval range, and terrain conditions. The tangent vectors at several points are specified and allow for directional control of the CoM through the terrain. The plot on the right shows curves generated at various terrain points. The final CoM path is constructed by connecting the minimum length OGH/COH curve at each interval range. Note that the freedom to select the endpoint CoM positions and tangent angles endows the planner with the flexibility and power to plan complex yet smooth CoM paths over any terrain.

A. Contact transition

The complete CoM path computed above is chosen as the preselected geometric path. With the path in place, we can use the numerical integration techniques of section III-C to derive the state-space behavior of the CoM for given foot contact locations and CoM velocities. Fig 3 depicts phase diagrams of CoM behavior for an example OGH curve CoM path and various boundary conditions, namely, the CoM velocities over the contact foot location. Note that the state-space behavior of the CoM depends on the sagittal and vertical placement of the foot contact location, as well as the apex velocity of the CoM as it passes over the foot location. As will be discussed shortly, this important consequence can be exploited to optimize the location of foot placements within a terrain. For the nonce, we observe that the predicted phase curves of CoM sagittal behavior can be used to find the step transitions between foot placements. This is done by finding the intersections of adjacent sagittal phase curves [17]. This ensures continuity of position and velocity between adjacent CoM behaviors. To determine the exact step transition point, we fit a 6th degree polynomial to the sagittal CoM phases for adjacent foot placements. The two polynomials are subtracted and the roots are found, expunging any imaginary roots. The point of intersection is then obtained by selecting the root that lies within the CoM

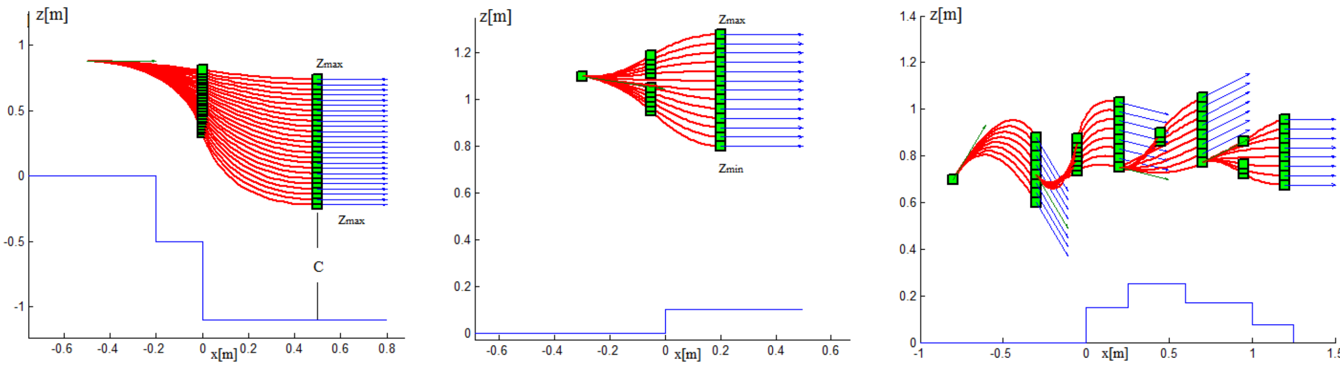

Fig. 2. CoM paths generated using COH curves for different CoM range constraints over various terrains. The left and middle plot demonstrate how the environment and the selection of tangent angles affects the CoM paths. The right plots displays a complete CoM path consisting of piecewise OGHCOH curves with minimum curvature and length to traverse the terrain.

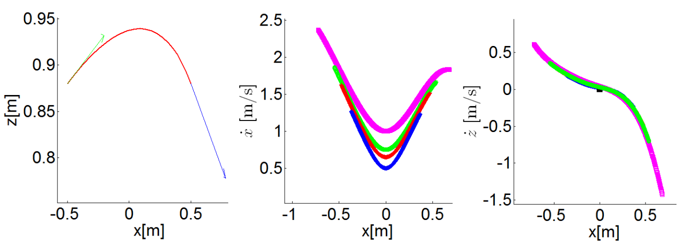

Fig. 3. OGH curve with $\theta = 10$ and $\phi = 340$. The phase diagrams of sagittal and vertical CoM velocities are shown in the middle and right plots respectively. The CoP is located at $(0, 0)$. Various boundary apex CoM velocities are shown.

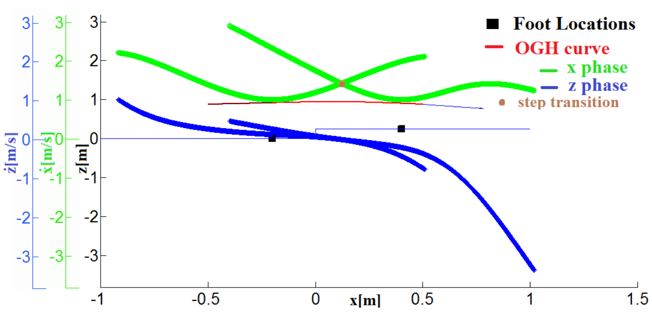

Fig. 4. CoM saggital and vertical phase diagrams for two contact feet at $(-0.2, 0)$ and $(0.4, 0.25)$ with the step transition point shown. The apex velocity of the CoM over each contact point is $1m/s$.

range. Fig 4 demonstrates the contact transition model for the OGH curve of Fig 3 with inclusion of a secondary foot location and terrain. For each contact foot location, the CoM sagittal phase space behavior is shown directly above it in green. The contact transition is then the point of intersection of adjacent sagittal phase curves. Naturally, shifting the foot contact location alters the resultant CoM state-space behavior and transition model. Perforce, prudent selection of foot contact placements is required as it will have considerable impact on the overall performance of the system. Lastly, it is consequential to note that the switching time in the contact transition between the x phase and the z phase as shown in Fig 4 is guaranteed to be synchronized. This is because the preselected CoM path(OGH curve) creates a dependency between the vertical CoM position and sagittal CoM position. The vertical CoM position is a function of sagittal CoM position thus ensuring that the contact transition for both occurs simultaneously.

B. Foot Placement Optimization

Previously, we used OGH curves to plan and ensure the smoothness of the CoM path over terrain. However, this alone is insufficient as the velocities of the CoM are still susceptible to extreme changes. This is especially evident when traversing over rugged and uneven terrain that cause *jerkiness* in the velocity profiles of the CoM, especially the CoM vertical phase. Indeed, an appealing quality of animal and human locomotion is the ability to plan and perform complex actions and maneuvers in smooth and steady manners, oftentimes in severely rugged environments. In our case, the most sensitive factor prone to *jerkiness* is the CoM vertical phase behavior as it is more adversely affected by the terrain. Thus, we now turn our attention to diminishing the negative impact of the up and down motions of the terrain on the CoM vertical phase behavior.

V. Simulation Results

We used Matlab to generate simulation results to study vertical phase behavior and methods to temper vertical jerkiness. Earlier we noted the effect of foot placements on CoM phase behavior. Moving a contact foot location up, down, left, or right causes changes not only in CoM sagittal and vertical phase behavior, but also in the step transition point of the foot with its nearest neighbors. Fig 5 below depicts two example terrains and CoM paths and the effects on CoM phase behavior resulting from changing the secondary foot contact location. Additionally, the CoM apex velocity over the contact foot location also serves to affect the phase space behavior of the CoM. In this study we treat the CoM apex velocity as constant from contact foot to contact foot as that is a natural choice for traveling through environments. From Fig 4 we observe that the complete vertical phase of the CoM for adjacent foot contacts is composed of the forward vertical phase segment for the back foot starting at its contact location, $p_{cop_{x1}}$, up to and including the step transition point

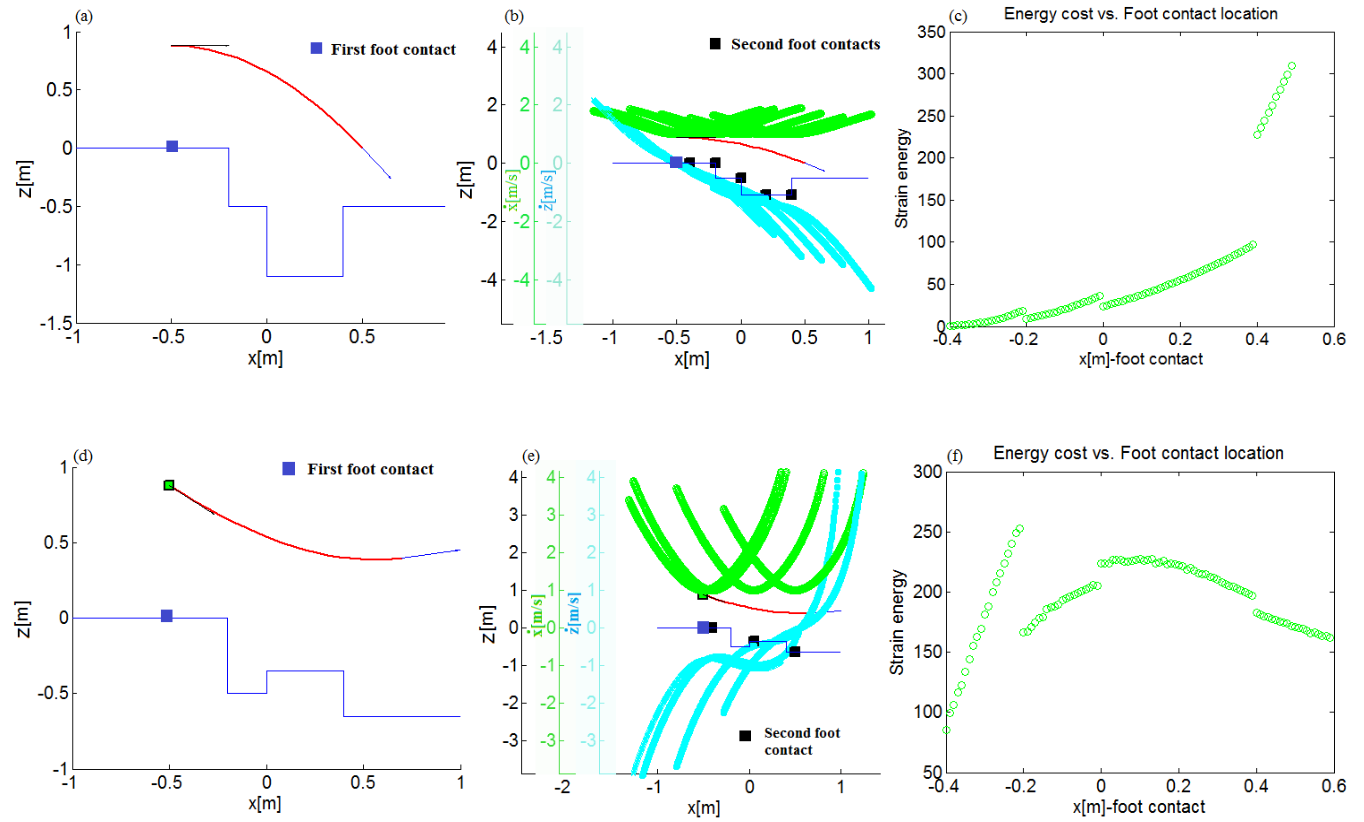

Fig. 5. OGH CoM paths with constant apex velocity $v = 1m/s$ and distinct tangent and position endpoint conditions traversing different terrains are shown in (a) and (d). In (b) and (e) several second foot contact locations and the corresponding phase space behavior of the CoM are shown demonstrating the affects of foot locations. The minimum energy cost landscape for each terrain is shown in (c) and (f) respectively.

x_{tp}, and the backward vertical phase segment for the front foot starting at its contact location, $p_{cop_{x2}}$ backwards to x_{tp}. To gain insight into the vertical phase behavior of the CoM, we fit a 5th degree polynomial to the aforementioned curve and define it in the range $(p_{cop_{x1}}, p_{cop_{x2}})$. Let this curve be $h(x)$. Thus, we seek foot placements within the environment such that $h(x)$ is optimized to reduce its volatility. To do this, we use the approximate strain energy for the curve(also called the linearized bending energy) defined as:

$$S = \int_{x_0}^{x_1} [h''(x)]^2 dx \tag{10}$$

which returns a scalar that quantifies the degree of bending in the curve. It is precisely the bending and undulating nature of the CoM vertical phase that we seek to diminish. Therefore, we seek foot contact placements that minimize this quantity. To obtain the minimum strain energy vertical phase curve, we integrate $f(x)$ from $x_0 = p_{cop_{x1}}$ for each $x_1 = p_{cop_{x2}}$ where $p_{cop_{x2}} \in p_{ter_x}$. This is equivalent to iterating through the terrain by placing the secondary foot contact location at each point in the environment and calculating its strain energy. This produces an energy cost landscape from which the minimum strain energy contact foot location can be extrapolated based on the kinematic dynamics of the robot. Because each robot configuration has different kinematic constraints, selection of foot placements will vary from robot to robot. This process can subsequently be used to plan foot locations and step transitions through the environment. From Fig 5 we observe that the terrain enforces restrictions on the space of available foot contact locations as can be seen in (b) and (d). Furthermore, if the robot's kinematic constraints are included, then the space of potential foot placements is further reduced to fall within a range defined by that particular robot. Ergo, the minimum energy cost landscape can be used in conjunction with the robot's configuration to plan optimal foot placement strategies. For example, if the terrain and CoM path from Fig 5(a) were used with a robot with initial foot placement at $(-0.5, 0)$ and kinematic limits for maximum foot placement of $0.35m$, then an efficient choice for optimizing progress and minimizing vertical phase velocity disturbance would be to place the ensuing foot at $(-0.19, -0.5)$. Similarly, starting from that foot, the process can be repeated to generate the new energy cost landscape and plan yet another foot location. Naturally, situations may arise where some of the optimal foot placements will be difficult to access and may be unreachable for the robot. However, this can be rectified by introducing additional robot kinematics constraints into the framework. Example full step sequences for the terrains and paths of Fig 5 are shown in Fig 6. In each, the minimum strain energy cost landscape is derived starting from the preceding foot to next potential foot locations. This produces a guide for selection of subsequent optimal foot contact locations. To examine

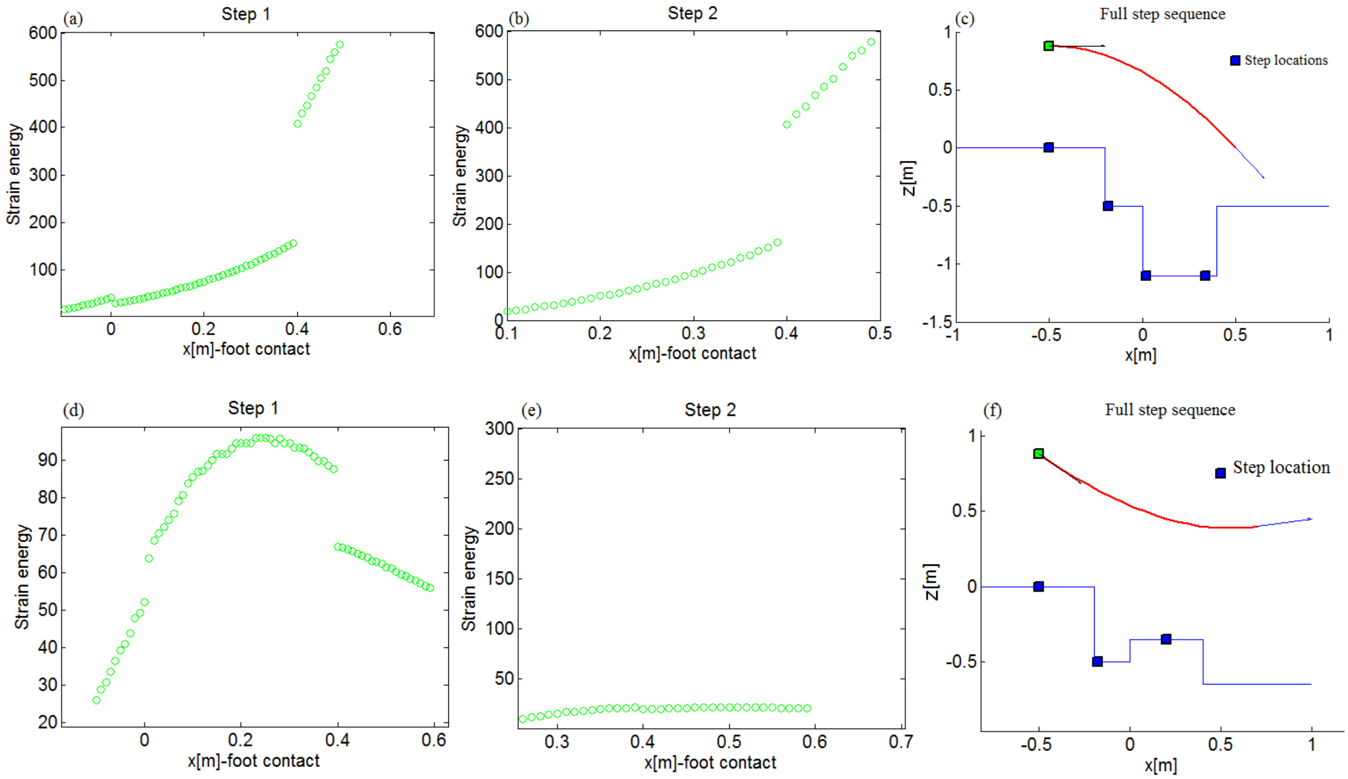

Fig. 6. Enery cost landscape for CoM paths and terrains of Fig 5 at step 1 and step 2 shown in (a-b) and (d-e) respectively. The cost is the minimum strain energy of the vertical CoM phase between adjacent potential foot locations. The maximum foot placement for each robot is $0.35m$ and $0.4m$. (c) and (f) are the full step sequences.

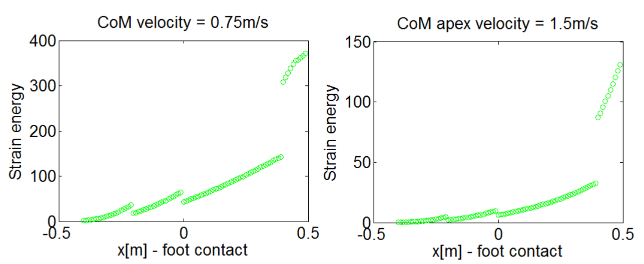

Fig. 7. Minimum energy cost landscape for Fig. 5a for CoM constant apex velocities of $v = 0.75m/s$ and $v = 1.5m/s$.

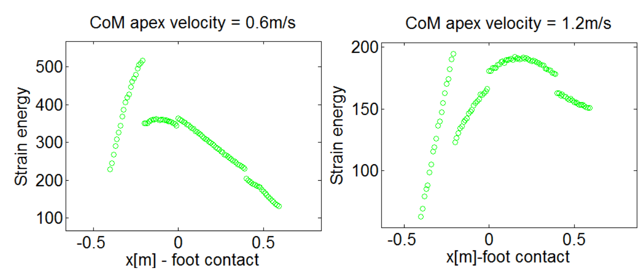

Fig. 8. Minimum energy cost landscape for Fig. 5d for CoM constant apex velocities of $v = 0.6m/s$ and $v = 1.2m/s$.

contact foot selection further, we are currently generating larger simulation results and generalizing to multi-contact dynamics to test our methods on Atlas to simulate dynamics of real robots.

Needless to say, the performance of the system is also ineluctably tied to the choice of CoM apex velocity, and careful attention will have to be taken in its selection. Nevertheless, it is the power and versatility of this framework that despite the choice of velocity, foot placement locations and step transitions can still reliably be obtained such that the robot stays within its kinematic constraints. Fig 7 and Fig 8 show the energy cost landscapes that arise under various CoM apex velocities. Note that increased velocities tend to reduce the cost of foot placements in the terrain, whereas decreased velocities magnify them. This agrees with intuition since slower velocities tend to exacerbate the conditions in the environment, whereas faster velocities tend to diminish the effects. However, the important consequence is that despite the velocity requirements for the robot, foot placement and transition plans are derivable that ensure the robot traverses the terrain in a manner that minimizes the effects on the CoM phase behavior while traveling along a smooth CoM path. Thus, in conjunction with the CoM paths, the energy cost landscapes can always be used to plan optimal foot contact locations through the environment.

VI. Conclusion

Robot locomotion planning over terrains is difficult due to the coarseness of environments and the lack of representational models that incorporate the variance of kinematic shapes and sizes present in robots. In this paper we sought to encapsulate these differences by creating abstractions that allowed us to stably and reliably plan CoM paths for a robot over a terrain. We then used this as a springboard to plan optimal foot placements in the environment. In particular, assigning smooth paths free of irregularities is imperative as the environment is subject to extreme changes. To this end, we planned minimum curvature paths through the terrain using OGH curves that respect the robot's kinematic constraints. With these paths in place, we then used the single contact dynamic model to study the phase space behavior of the CoM by varying the locations of adjacent foot contacts. The potential foot contact locations depend on the terrain and have significant effects on the CoM vertical phase behavior. To attenuate the effects on the CoM vertical phase, we used Matlab simulation studies and the minimum strain energy between potential contact points to create cost landscapes. In conjunction with the kinematic constraints of the robot, these cost landscapes allowed us to select optimal foot locations through the environment.

Naturally, sequential foot contacts tend to overlap and vary in duration based on the motion of the robot and the demands placed on it by the environment. We are currently incorporating multi-contact dynamics and testing our framework on Atlas to examine the behavior of systems with multiple contact points impinging the terrain. This will allow us to generalize further and study more complicated motions requiring simultaneous foot contacts. Additionally, constraining the CoM apex velocity to be constant from contact point to contact point may serve to limit selection of optimal foot placements. Most animals shift and alter their velocities depending on terrain. Hence, we plan on investigating the effects of changing CoM apex velocities to add increased versatility and flexibility into our framework. Together with multi-contact dynamics, the resultant behavior will more closely resemble the extreme locomotion maneuvers observed in nature.

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